

Complexity guarantees and polling strategies for Riemannian direct-search methods

Bastien Cavarretta, *F. Goyens, C. Royer and F. Yger*

JKU Linz | 9. Juli 2026

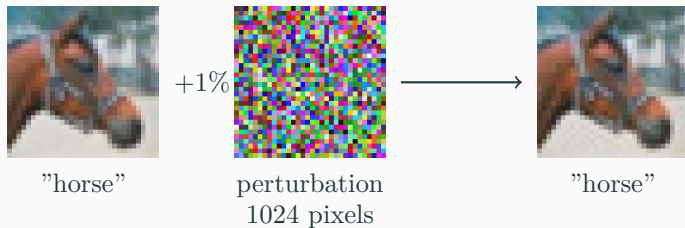
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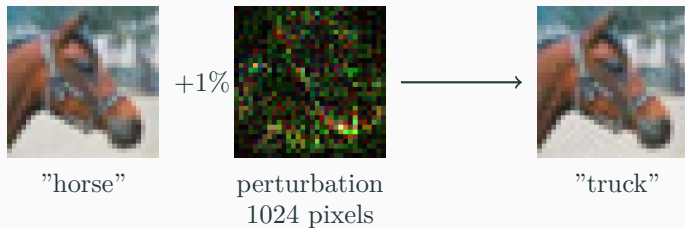


Motivation: Adversarial attacks of Neural Networks



Goal: Fool a classifier $g : \text{images} \rightarrow \text{label}$.

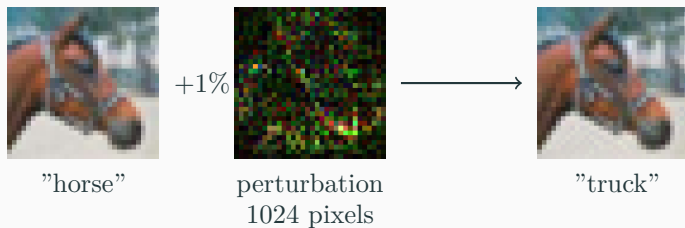
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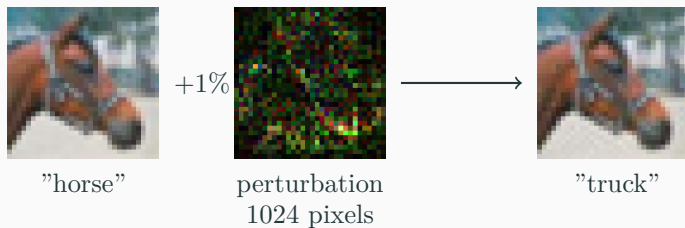


Goal: Fool a classifier $g : \text{images} \rightarrow \text{label}$.

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$$\max_{\text{small}} \text{ce}(g(\text{horse} + \text{perturbation}), \text{"horse"})$$

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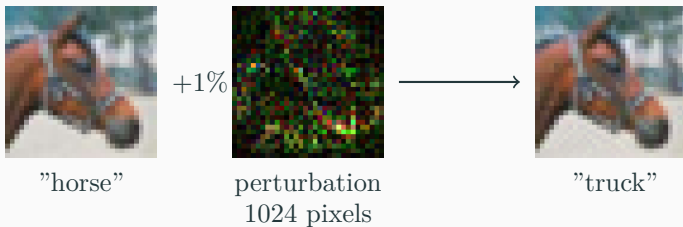


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$$\max_{\substack{\delta \in \mathbb{R}^{1024 \times 3} \\ \|\delta\|_2 \leq 0.5}} \text{ce}(g(x + \delta), y)$$

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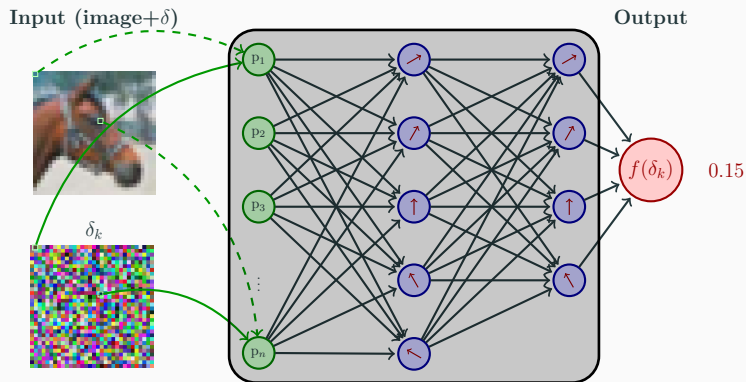


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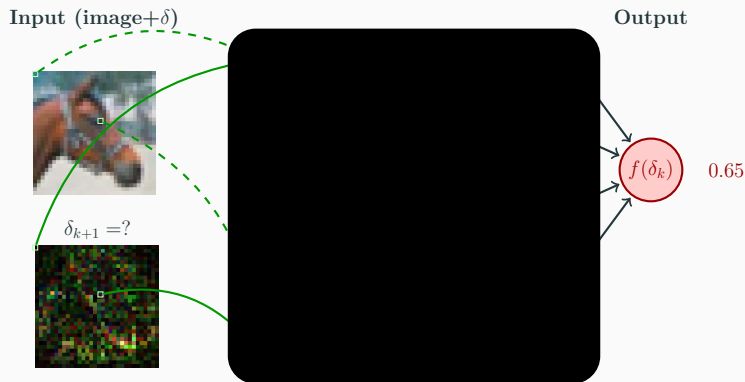
$$\max_{\substack{\delta \in \mathbb{R}^{1024 \times 3} \\ \|\delta\|_2 \leq 0.5}} f(\delta)$$

Black-box attacks (1/2)



Whitebox attack (gradients ✓): $\delta_{k+1} = \delta_k + \gamma \nabla_{\delta} f(\delta_k)$

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Black-box attack (gradients ✗): $\delta_{k+1} = ?$

→ **Derivative-free optimization**

Black-box attacks (2/2)

Need for scalable methods

- CIFAR10 : dimension 1024×3
- Subspace approaches (Chen et al., ZOO, '17)
- More fancy attacks : Wavelet domain (Cai et al. '21).
Dimension $> 10^6$.

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- Probabilistic Direct Search (Gratton et al. '15)
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- Manifold direct search (Dreisigmeyer '06 '07 '18 '19),
(Kungurtsev et al. '24)

$$\min_{x \in \mathcal{M}} f(x)$$

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- f smooth, black-box, bounded below.

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Our work

- Complexity bound for direct search
- Guidelines for implementing polling directions

Riemannian Direct Search and Complexity

Projected or intrinsic polling

Numerical experiments

Riemannian Direct Search and Complexity

Riemannian direct-search

Input $x_0 \in \mathbb{R}^m$, $0 < \alpha_0 < \alpha_{\max}$, $c > 0$.

For $k = 1, 2, \dots$

- Take a PSS $D \subset \mathbb{R}^m$.
- If there exists $d \in D_k$ s.t. $f(x_k + \alpha_k d) \leq f(x_k) - c\alpha_k^2 \|d\|^2$,
 $x_{k+1} = x_k + \alpha_k d$, $\alpha_{k+1} = \min(2\alpha_k, \alpha_{\max})$.
- Else, $x_{k+1} = x_k$, $\alpha_{k+1} = \frac{1}{2}\alpha_k$.

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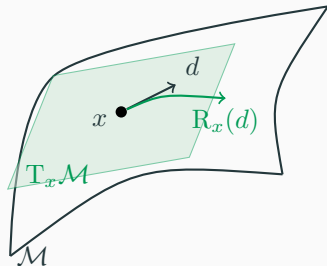
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What changes

$T_x \mathcal{M}$: Tangent space of $\mathcal{M}$ at x .

$\mathbf{R}_x(d)$: Retraction, move on the manifold with initial speed d .



Convergence of Riemannian direct-search

Asymptotic convergence (Kungurtsev, Rinaldi, Zeffiro '24)

If $\text{grad}f$ is L -Lipschitz and directions D_k are well-chosen, then,

$$\lim_{k \rightarrow \infty} \|\text{grad}f(x_k)\|_{x_k} = 0$$

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- D_k computed by projecting PSSs of $\mathbb{R}^n$ into $T_x\mathcal{M}$.

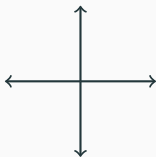
What is a *good set* of directions in $\mathbb{R}^m$?

Positive Spanning Set (PSS) of $\mathbb{R}^m$:

Set $D \subset \mathbb{R}^m$ s.t, $\text{cone}(D) = \mathbb{R}^m$.

Cosine measure of D :

$$\text{cm}(D) := \min_{\substack{v \in \mathbb{R}^m \setminus \{0\}}} \max_{\substack{d \in D \\ d \neq 0}} \frac{\langle v, d \rangle}{\|v\| \|d\|}.$$

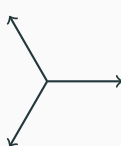


D

$[I_m, -I_m]$

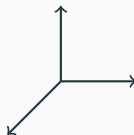
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unif. angles

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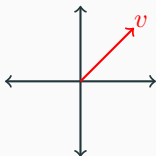
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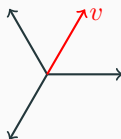


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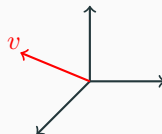
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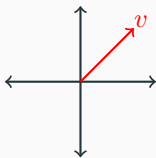
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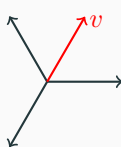


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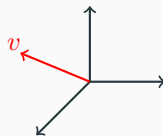
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Euclidean direct-search complexity

Context: $\min_{\mathbb{R}^m} f$ with fixed PSS D (Euclidean)

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Theorem (Vicente '12)

If ∇f is L -Lipschitz, the method reaches $\|\nabla f(x_k)\| \leq \epsilon$ in less than

$$O(|D|\text{cm}(D)^{-2} \times \epsilon^{-2}) \text{ function evaluations}$$

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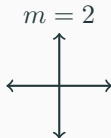
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- $|D^\oplus| = 2m$
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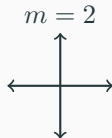
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Theorem (Dodangeh, Vicente, Zhang '15)

$\chi(D)$ cannot be better than $O(m^2)$ (optimal order of WCC).

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Positive spanning sets on a manifold $\mathcal{M}$

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Generalizing to the whole manifold

Let $\mathcal{D} = (D_x)_{x \in \mathcal{M}}$, $D_x \in T_x\mathcal{M}$.

Cosine Measure of $\mathcal{D}$:

$$\text{cm}(\mathcal{D}) = \inf_{x \in \mathcal{M}} \text{cm}_x(D_x)$$

$\mathcal{D}$ is a PSS for $\mathcal{M}$ if, $\text{cm}(\mathcal{D}) > 0$.

Riemannian direct-search complexity

Context: $\min_{x \in \mathcal{M}} f$.

Assume:

- $\mathcal{D} = (D_x)_{x \in \mathcal{M}}$ PSS of $\mathcal{M}$.
- $D_k := D_{x_k}$ for every k .
- $f \geq f_{\text{low}}$ and $\text{grad} f$ Lipschitz continuous.

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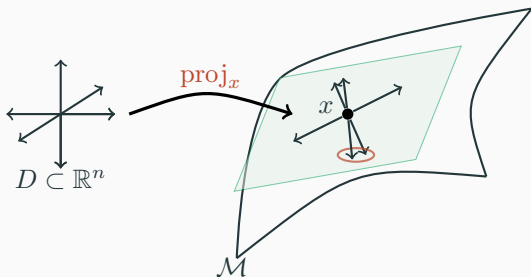
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Projected or intrinsic polling

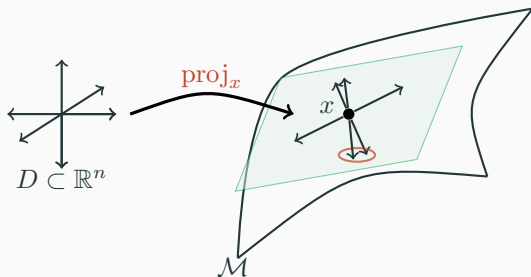
PSS for manifolds: Projected way

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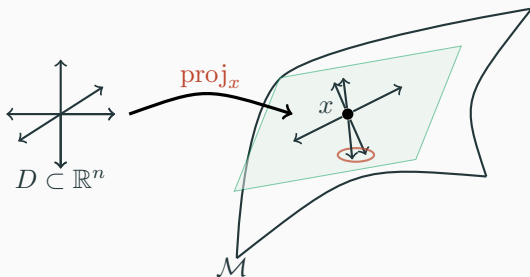


Projected PSS: Denote $P_x(D)$ projection of D onto $T_x\mathcal{M}$,

$$\mathcal{P}(D) := (P_x(D))_{x \in \mathcal{M}}.$$

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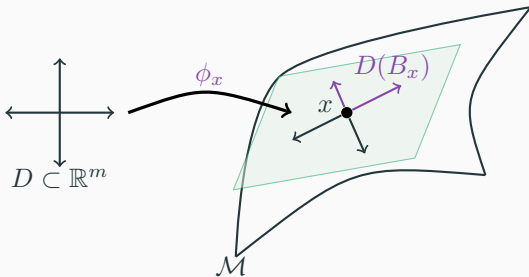
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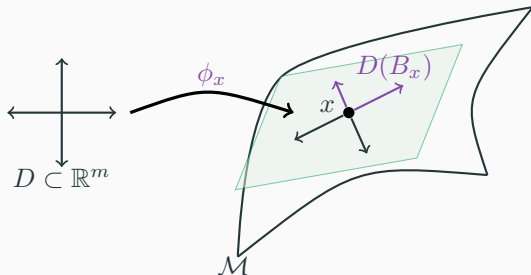
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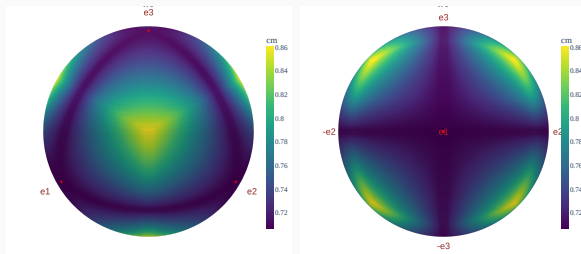


Intrinsic PSS: From D PSS of $\mathbb{R}^m$ and $\mathcal{B} = (B_x)_{x \in \mathcal{M}}$ bases, build

$$D(\mathcal{B}) := (D(B_x))_{x \in \mathcal{M}}.$$

$$\text{cm}_{\mathcal{M}}(D(\mathcal{B})) = \text{cm}(D), \quad \chi_{\mathcal{M}}(\mathcal{P}(D)) = \chi_m(D)$$

A simple case: $\mathcal{M} = \mathbb{S}^{n-1} := \{x \in \mathbb{R}^n \mid \|x\|^2 = 1\}$



heatmap of $\text{cm}_{T_x \mathcal{M}}(P_x(D^\oplus))$

	Projected	Intrinsic
PSS	$\mathcal{P}(D^\oplus)$	$D^\oplus(B)$
size ℓ		$2(n-1)$
cm		$\frac{1}{\sqrt{n-1}}$
χ		$2(n-1)^2$

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Theorem (C., Goyens, Royer, Yger '25+)

On $\mathcal{M} = \mathbb{S}^{n-1}$,

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χ	$2n(n-1)$	$2(n-1)^2$

Numerical experiments

Positive spanning sets in $\mathbb{R}^m$

PSS	$D^\oplus$	$D^\ominus$	$D^\otimes$
			
$ \cdot $	$2m$	$m + 1$	$m + 1$
$\text{cm}(\cdot)$	$\frac{1}{\sqrt{m}}$	$\frac{1}{\sqrt{m^2 + 2(m-1)\sqrt{m}}}$	$\frac{1}{m}$
$\chi(\cdot)$	$2m^2$	$m^3 + O(m^{5/2})$	$m^3 + O(m^2)$
	 $D^\oplus$ (intr)  $D^\oplus$ (proj)	 $D^\ominus$ (intr)  $D^\ominus$ (proj)	 $D^\otimes$ (intr)  $D^\otimes$ (proj)

Toy manifold problems

Problems

Manifold	$f(x)$	Parameters
$F := \text{span}(z_1, \dots, z_m)$	$\sum_{i=1}^{10} \ x_i - x\ _2^2$	$(x_i) \in (\mathbb{R}^n)^{10}$
	$\sum_{i=1}^{10} \ x_i - x\ _2^2$	$(x_i) \in F^{10}$
	$\frac{1}{2} \langle Ax, x \rangle - \langle b, x \rangle$	$A \succcurlyeq \frac{1}{10} I_n, b \in \mathbb{R}^n$
$\mathbb{S}^{(m+1)-1} \times \{0_{\mathbb{R}^{n-m}}\}$	$\langle x, Ax \rangle$	$A \in \text{Sym}(n)$

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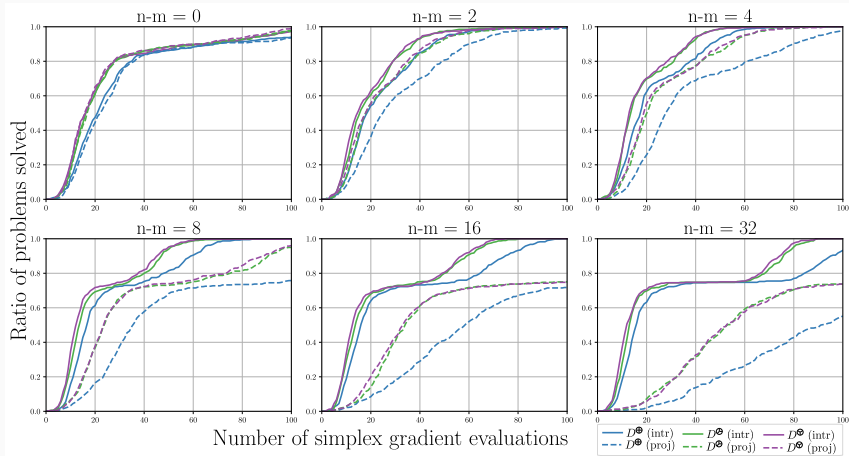
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Manifold	$f(x)$	Parameters
$F := \text{span}(z_1, \dots, z_m)$	$\sum_{i=1}^{10} \ x_i - x\ _2^2$	$(x_i) \in (\mathbb{R}^n)^{10}$
	$\sum_{i=1}^{10} \ x_i - x\ _2^2$	$(x_i) \in F^{10}$
	$\frac{1}{2} \langle Ax, x \rangle - \langle b, x \rangle$	$A \succcurlyeq \frac{1}{10} I_n, b \in \mathbb{R}^n$
$\mathbb{S}^{(m+1)-1} \times \{0_{\mathbb{R}^{n-m}}\}$	$\langle x, Ax \rangle$	$A \in \text{Sym}(n)$

Data profiles

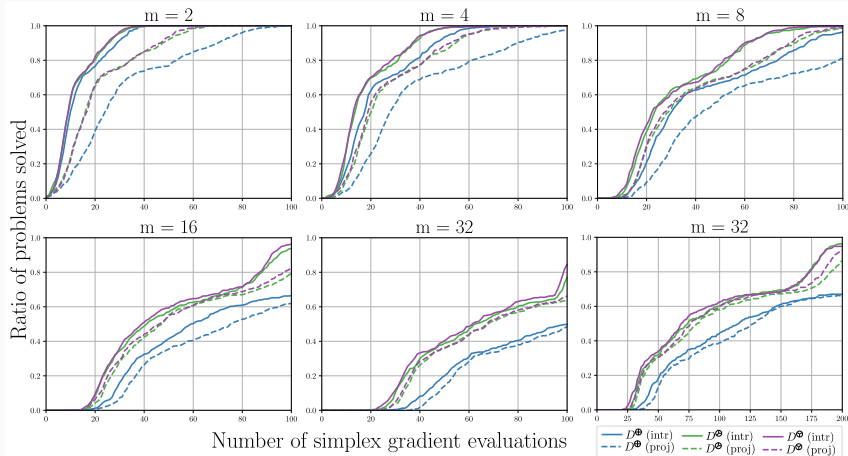
- 100 instances of each problem.
- Budget: $100(m+1)$ evaluations.
- Solved as soon as $\frac{f(x) - f_{\text{low}}}{f(x_0) - f_{\text{low}}} \leq 10^{-2}$.

Toy manifold problems



- Fixed $m = 4$. Varying $n - m$.
- Intrinsic PSSs outperform projected PSSs as $n - m$ grows.

Toy manifold problems



- Fixed $n - m = 4$. Varying m .
- Performance discrepancy fades as $\frac{n-m}{m} \rightarrow 0$.

Synchronization of rotations

$$\min_{R_1, R_2 \in \text{SO}(5)} \|R_1 - H_{12}R_2\|_F^2$$

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- $\mathcal{M} = \text{SO}(5) \times \text{SO}(5) \rightarrow m = 20, n = 50.$
- $T_{(R_1, R_2)} \mathcal{M} = R_1 \text{Skew}(5) \times R_2 \text{Skew}(5)$

Synchronization of rotations

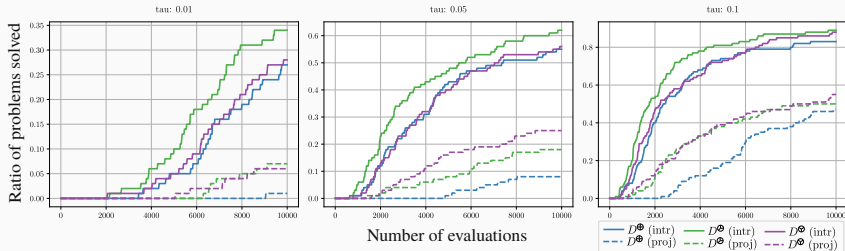
$$\min_{R_1, R_2 \in \text{SO}(5)} \|R_1 - H_{12}R_2\|_F^2$$

- $\mathcal{M} = \text{SO}(5) \times \text{SO}(5) \rightarrow m = 20, n = 50$.
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- Budget: $500(m + 1)$ evaluations.
- 100 problem instances (various $H_{12} \in \mathbb{R}^{5 \times 5}$)
- Solved as soon as $\frac{f(x) - f_{\text{low}}}{f(x_0) - f_{\text{low}}} \leq \tau, \tau \in \{0.01, 0.05, 0.1\}$.

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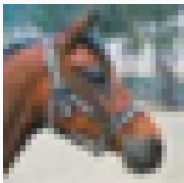


Intrinsic PSSs (plain) still outperform projected PSSs.

Black-box adversarial attacks

Setup

- Network: Wide ResNet 28-10.
- Victims: 90 images from CIFAR10.



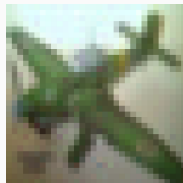
“truck”



“bird”



“frog”

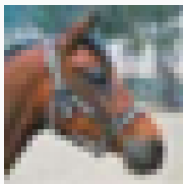


“frog”

Black-box adversarial attacks

Setup

- Network: Wide ResNet 28-10.
- Victims: 90 images from CIFAR10.
- Perturbation: low-rank matrix $\delta = UV$ where $U, V^T \in \mathbb{R}^{32 \times r}$.
- Problem: $\max_{\|\delta\|_2 \leq 0.5} f(\delta)$ with $f(\delta) = \text{ce}(x + \delta)$



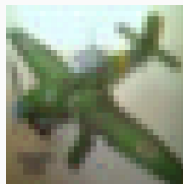
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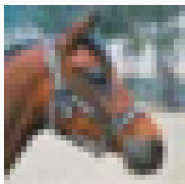


“frog”

Black-box adversarial attacks

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- Network: Wide ResNet 28-10.
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- Perturbation: low-rank matrix $\delta = UV$ where $U, V^T \in \mathbb{R}^{32 \times r}$.
- Problem: $\max_{\|\delta\|_2 \leq 0.5} f(\delta)$ with $f(\delta) = \text{ce}(x + \delta)$
- Algorithms: Direct-search (DS), Probabilistic version (DS-PD), finite difference gradient descent (GFM)
- Budget: 30 720 queries.
- **Dataprofiles:** problems solved if $\frac{f(\delta) - f(\delta_*)}{f(\delta_0) - f(\delta_*)} < \frac{1}{10}$



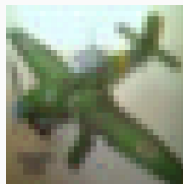
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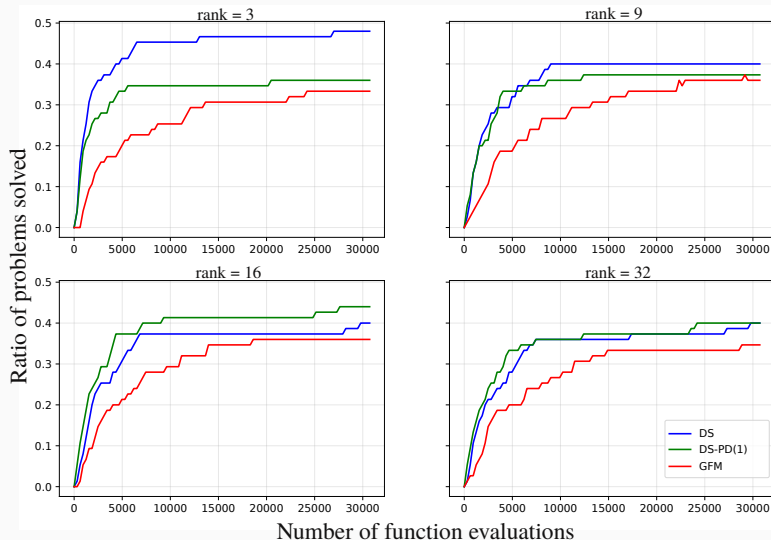


“frog”



“frog”

Black-box adversarial attacks



DS competes with finite-difference methods at equal budget

Our work

- Complexity analysis of Riemannian direct-search methods.
- Formalization of PSS for manifolds, advocate for intrinsics.

Direction

- More ambitious manifold application.
- Bound constraints $\longrightarrow$ Manifolds with corners ?

Vielen Dank



Mathematics > Optimization and Control

[Submitted on 19 Nov 2025 (v1), last revised 29 May 2026 (this version, v2)]

Complexity guarantees and polling strategies for Riemannian direct-search methods

Bastien Cavarretta, Florentin Goyens, Clément W. Royer, Florian Yger